Number Systems

Dr. Anish Soni
Assistant Professor
Dept. of Computer Applications

Number System

Bits and Bytes

- A single unit of data is called a bit, having a value of 1 or 0.
- Computers work with collections of bits, grouping them to represent larger pieces of data, such as letters of the alphabet.
- Eight bits make up one byte. A byte is the amount of memory needed to store one alphanumeric character.
- With one byte, the computer can represent one of 256 different symbols or characters.
- Computers store all data as binary digits, but we may need to convert this to a number system we are familiar with.

Number System

Two types of number systems are:

- Non-positional number systems
- Positional number systems

Non-positional Number Systems

Characteristics

- Use symbols such as I for 1, II for 2, III for 3, IIII for 4, IIIII for 5, etc in Uniary number system or I for 1, II for 2, III for 3, IV for 4, V for 5 in Roman Number System
- Each symbol represents the same value regardless of its position in the number
- The symbols are simply added to find out the value of a particular number

Difficulty

- It is difficult to perform arithmetic with such a number system

Positional Number Systems

Characteristics

- A **positional** (**numeral**) **system** is a **system** for representation of **numbers** by an ordered set of numerals symbols (called digits)
- Use only a few symbols called digits
- These symbols represent different values depending on the position they occupy in the number

Examples are:

- Decimal Number System
- Binary Number System
- Octal Number System
- Hexadecimal Number System

- When we type some letters or words, the computer translates them in numbers as computers can understand only numbers.
- A computer can understand positional number system where there are only a few symbols called digits and these symbols represent different values depending on the position they occupy in the number.
- A value of each digit in a number can be determined using the digit ,the position of the digit in the number and the base of the number system

Common Number Systems

System	Base	Symbols	Used by humans?	Used in computers?
Decimal	10	0, 1, … 9	Yes	No
Binary	2	0, 1	No	Yes
Octal	8	0, 1, … 7	No	No
Hexa-decimal	16	0, 1, … 9, A, B, … F	No	No

Decimal Number System

- The number system that we use in our day-to-day life is the decimal number system.
- Decimal number system has base 10 as it uses 10 digits from 0 to 9.
- In decimal number system, the successive positions to the left of the decimal point represent units, tens, hundreds, thousands and so on.
- Each position represents a specific power of the base 10.
- For example, the decimal number 1234 consists of the digit 4 in the units position, 3 in the tens position, 2 in the hundreds position, and 1 in the thousands position, and its value can be written as

$$(1\times1000)+ (2\times100)+ (3\times10)+ (4\times1)$$
$$(1\times10^3)+ (2\times10^2)+ (3\times10^1)+ (4\times10^0)$$
$$1000 + 200 + 30 + 4$$
$$1234$$

Binary Number System

- Uses two digits, 0 and 1.
- Also called base 2 number system
- Right most position in a binary number represents a 0 power of the base 2. Example 2^0
- Last position in a binary number represents a x power of the base 2. Example 2^x where x represents the last position
- Example Binary Number : 10101_2

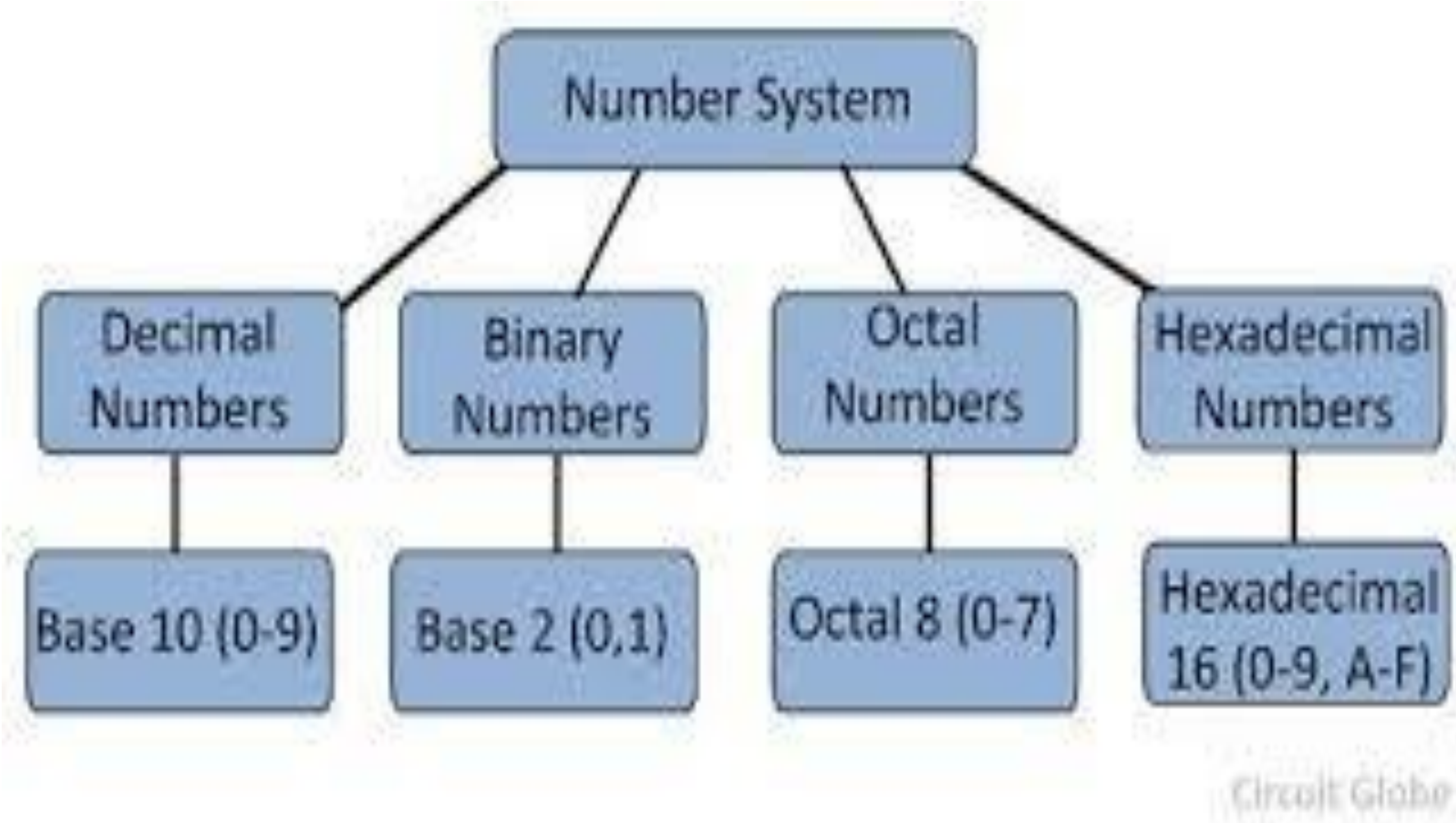
Octal Number System

- Uses eight digits, 0,1,2,3,4,5,6,7.
- Also called base 8 number system
- Right most position in a octal number represents a 0 power of the base 8. Example 8^0
- Last position in a binary number represents a x power of the base 8. Example 8^x where x represents the last position
- Example Binary Number : 1257_8

Hexadecimal Number System

- Uses 16 digits, 0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F.
- Letters represents numbers starting from 10.

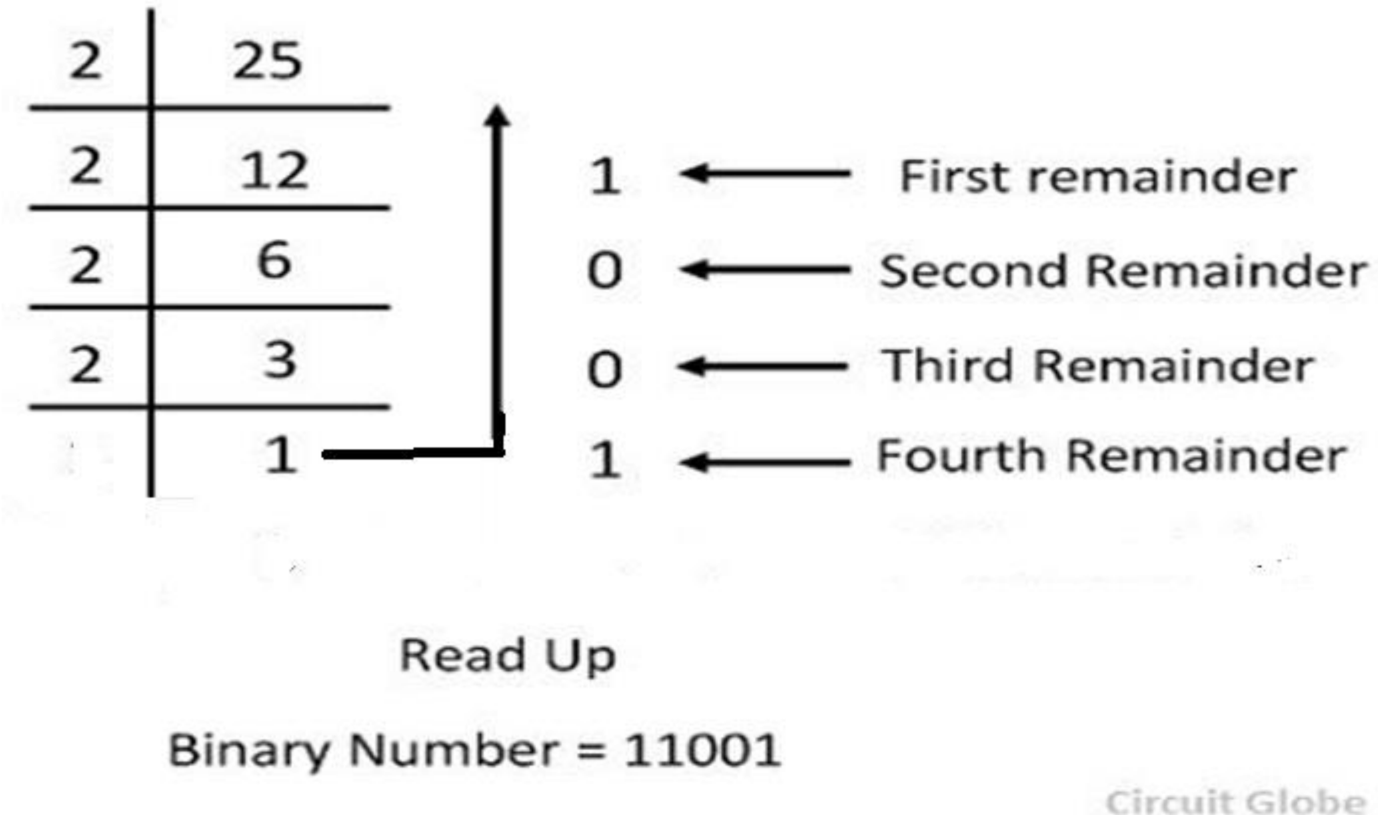
  A = 10, B = 11, C = 12, D = 13, E = 14, F = 15.
- Also called base 16 number system
- Right most position in a hexadecimal number represents a 0 power of the base 16. Example 16^0
- Last position in a binary number represents a x power of the base 16. Example 16^x where x represents the last position
- Example Binary Number : $19FD_{16}$


Number System
Decimal Numbers
Binary Numbers
Octal Numbers
Hexadecimal Numbers
Base 10 (0-9)
Base 2 (0,1)
Octal 8 (0-7)
Hexadecimal 16 (0-9, A-F)
Circuit Globe

Number Conversions

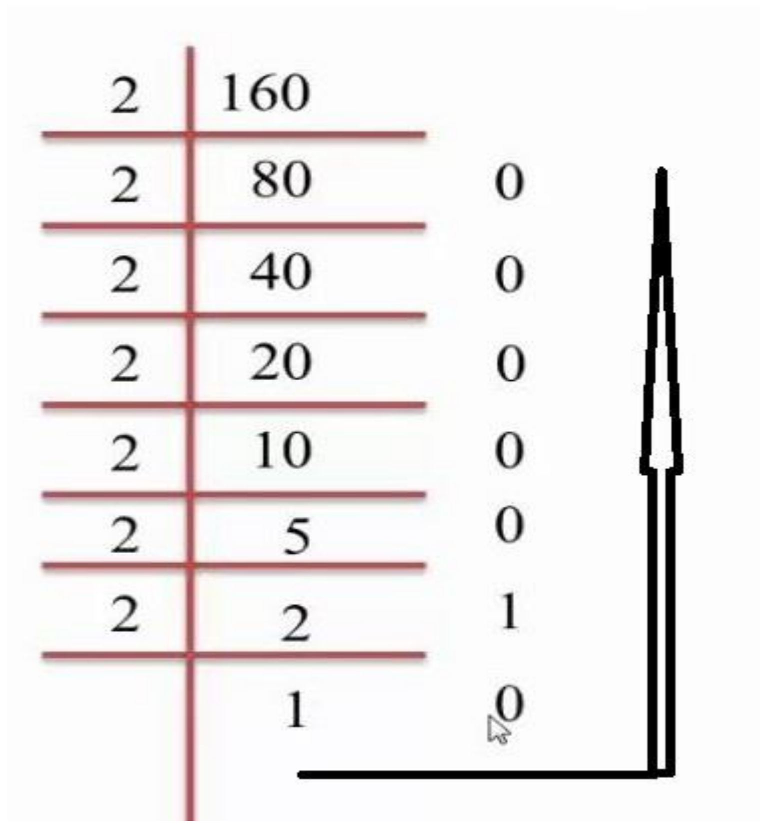
1. Decimal to Binary

- $(25)_{10}=(?)_2$



Decimal to Binary

- $(160)_{10}=(?)_2$



$$(160)_{10}=(10100000)_2$$

Convert $(131.25)_{10}$ to $(?)_2$

2	131	
2	65	1
2	32	1
2	16	0
2	8	0
2	4	0
2	2	0
	1	0

Read buttom to top

10000011

$.25 \times 2 = \underline{0}.5$

$0.5 \times 2 = \underline{1}.0$

Read top to buttom

01

Stop when it becomes 1 or figure start Repeating.

01

So $(131.25)_{10} = (10000011.01)_2$

$$(243.625)_{10} = (?)_2$$

$$
\begin{array}{r|l}
2 & 243 \\ \hline
2 & 121 - 1 \\ \hline
2 & 60 - 1 \\ \hline
2 & 30 - 0 \\ \hline
2 & 15 - 0 \\ \hline
2 & 7 - 1 \\ \hline
2 & 3 - 1 \\ \hline
 & 1 - 1
\end{array}
$$

11110011

$$0.625 \times 2 = \underline{1}.250$$

$$0.250 \times 2 = \underline{0}.5$$

$$0.5 \times 2 = \underline{1}.0$$

101

$$(243.625)_{10} = (11110011.101)_2$$

Convert $(17.325)_{10} = (?)_2$

2	17
2	8 – 1
2	4 – 0
2	2 – 0
	1 – 0

10001

$.325 \times 2 = 0.650$

$0.650 \times 2 = 1.300$

$0.3 \times 2 = 0.6$

$0.6 \times 2 = 1.2$

$0.2 \times 2 = 0.4$

$0.4 \times 2 = 0.8$

$0.8 \times 2 = 1.6$

$0.6 \times 2 = 1.2$

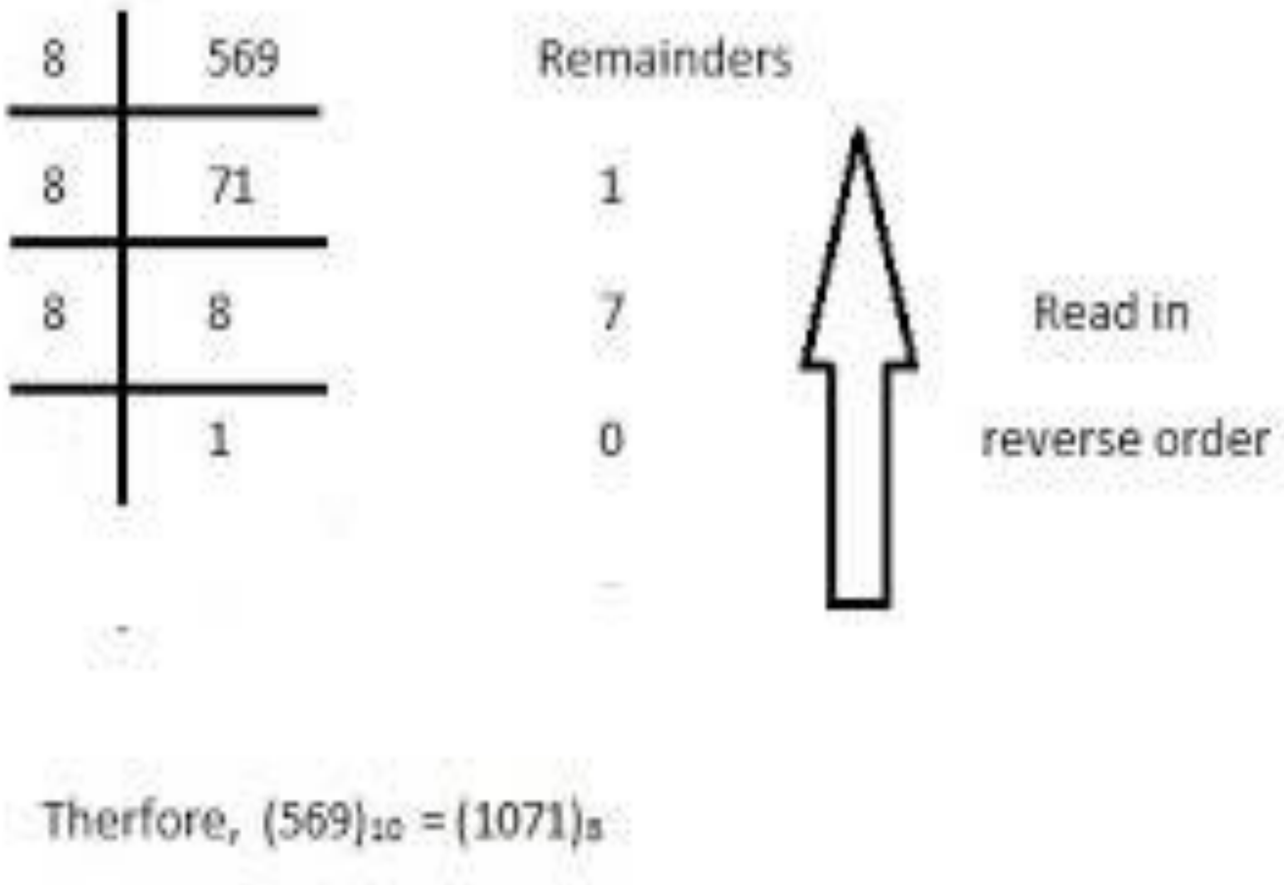
It starts repeating, so stop here

1010011

So $(17.325)_{10} = (10001.1010011)_2$

2. Decimal to Octal

- $(569)_{10}=(?)_8$



Convert $(235.56)_{10} = (?)_8$

8	235
8	29 – 3
	3 – 5

353

$0.56 \times 8 = \underline{4}.48$

$0.48 \times 8 = \underline{3}.84$

$0.84 \times 8 = \underline{6}.72$

$0.72 \times 8 = \underline{5}.76$

$0.76 \times 8 = \underline{6}.08$

You can quit after three or four points

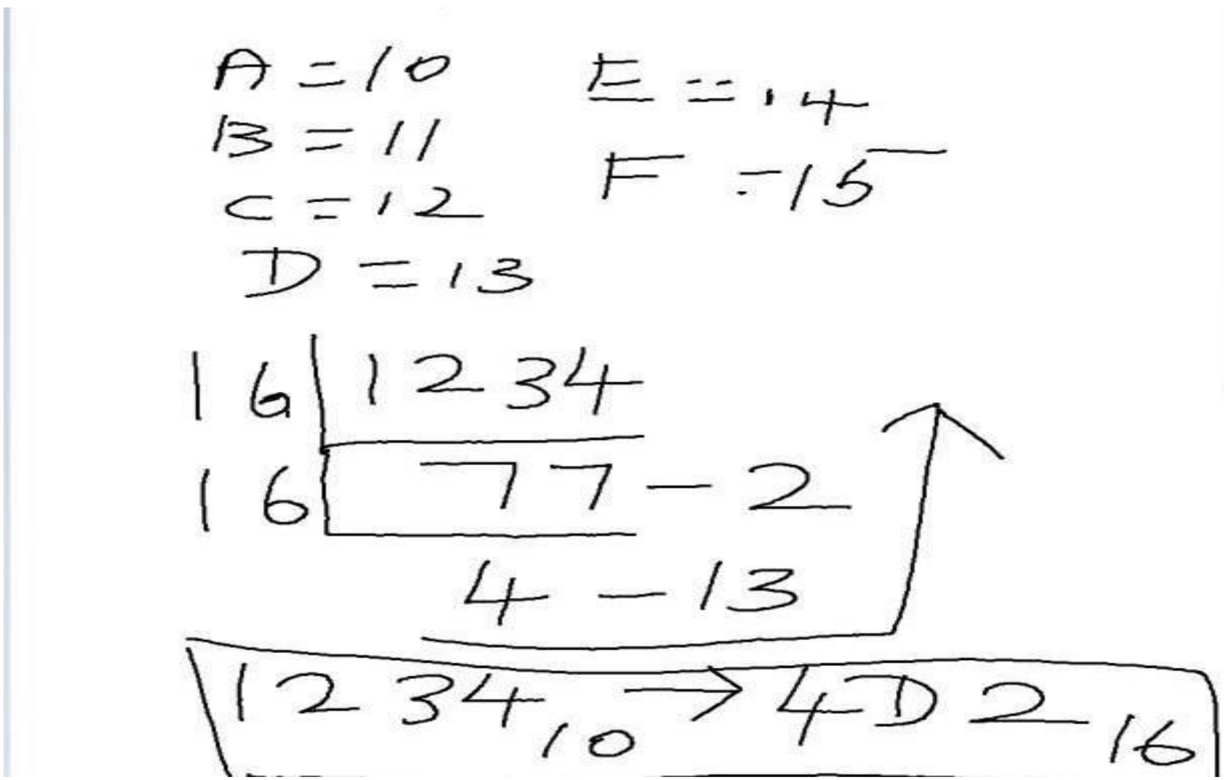
Read top to bottom (underline)

43656

So $(235.56)_{10} = (353.43656)_8$

3. Decimal to Hexadecimal

- $(1234)_{10}=(?)_{16}$



$(235.56)_{10} = (?)_{16}$

$$
\begin{array}{r|l}
16 & 235 \\
\hline
 & 14 - 11
\end{array}
$$

$14 \rightarrow E$

$11 \rightarrow B$

So EB

$0.56 \times 16 = \underline{8}.96$

$0.96 \times 16 = \underline{15}.36$

$0.36 \times 16 \quad \underline{5}.76$

$0.76 \times 16 \quad \underline{12}.16$

Quit

Now $15 \rightarrow F$

$12 \rightarrow C$

So Read from top to bottom

8F5C

$(235.56)_{10} = (EB.8F5C)_{16}$

Binary to Decimal Conversion

BINARY TO DECIMAL Technique

- Multiply each bit by 2^n, where n is the "weight" of the bit
- The weight is the position of the bit, starting from 0 on the right
- Add the results

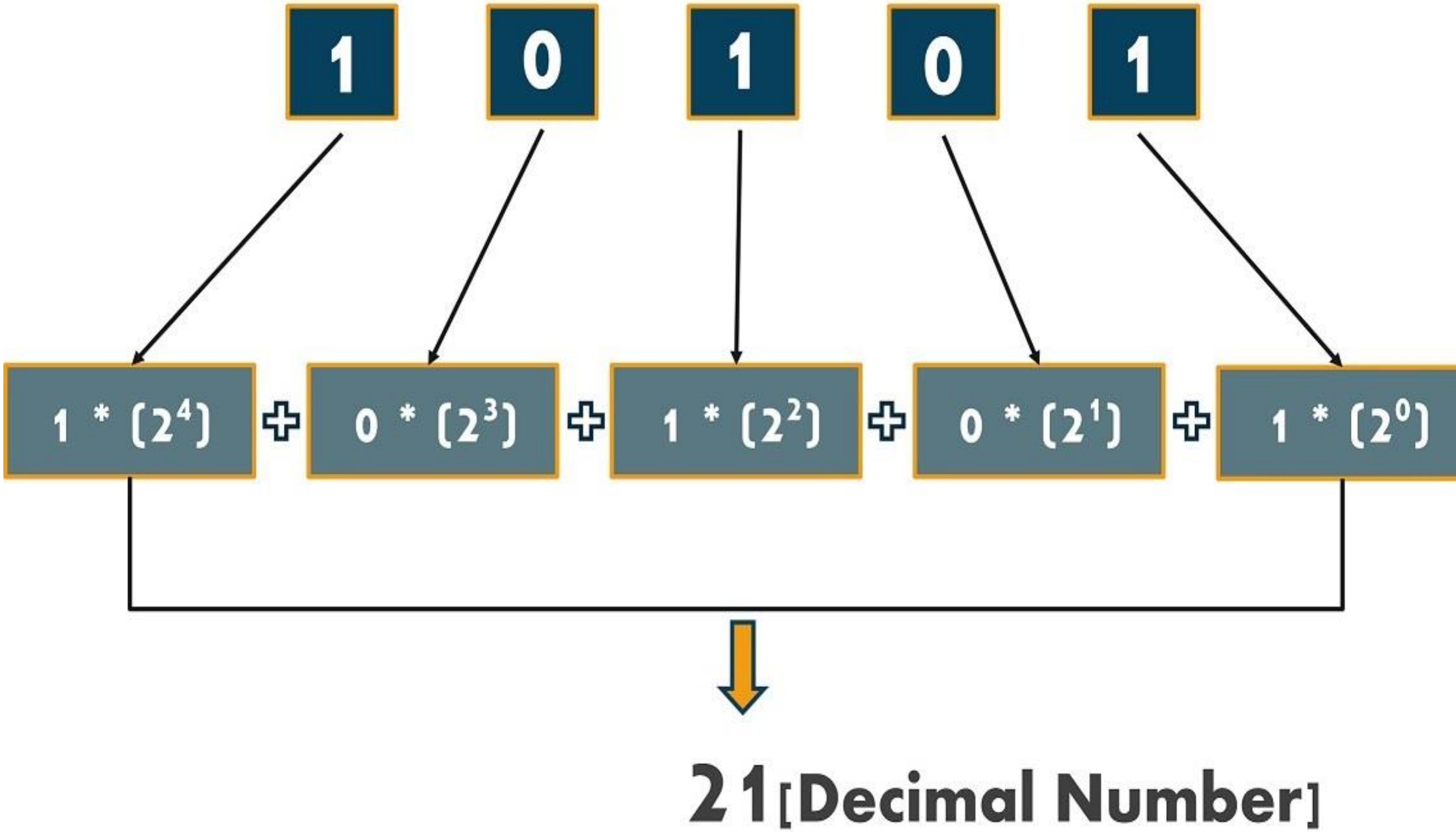
EXAMPLE

Bit "0"

$$101011_2 \Rightarrow$$

$$1 \times 2^0 = 1$$
$$1 \times 2^1 = 2$$
$$0 \times 2^2 = 0$$
$$1 \times 2^3 = 8$$
$$0 \times 2^4 = 0$$
$$1 \times 2^5 = 32$$

$$43_{10}$$


1
0
1
0
1
1 * (2^4)
+
0 * (2^3)
+
1 * (2^2)
+
0 * (2^1)
+
1 * (2^0)
21[Decimal Number]

Octal to Decimal Conversion

Octal TO DECIMAL Technique

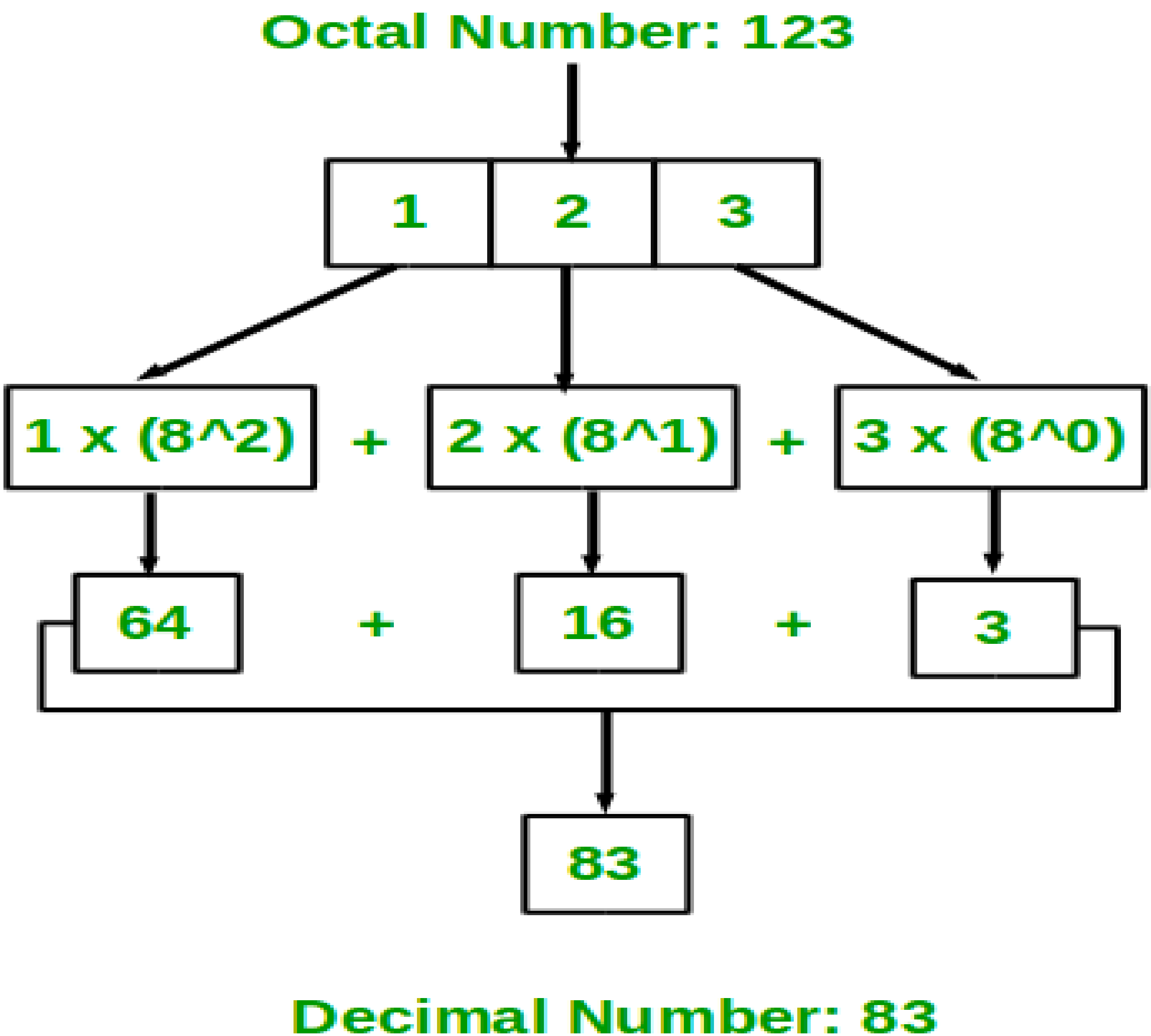
- Multiply each bit by 8^n, where n is the "weight" of the bit
- The weight is the position of the bit, starting from 0 on the right
- Add the results

EXAMPLE

$724_8 \Rightarrow$

$$4 \times 8^0 = 4$$
$$2 \times 8^1 = 16$$
$$7 \times 8^2 = 448$$

$$468_{10}$$



Octal Number: 123
1
2
3
1 x (8^2)
+
2 x (8^1)
+
3 x (8^0)
64
+
16
+
3
83
Decimal Number: 83

Hexadecimal to Decimal Conversion

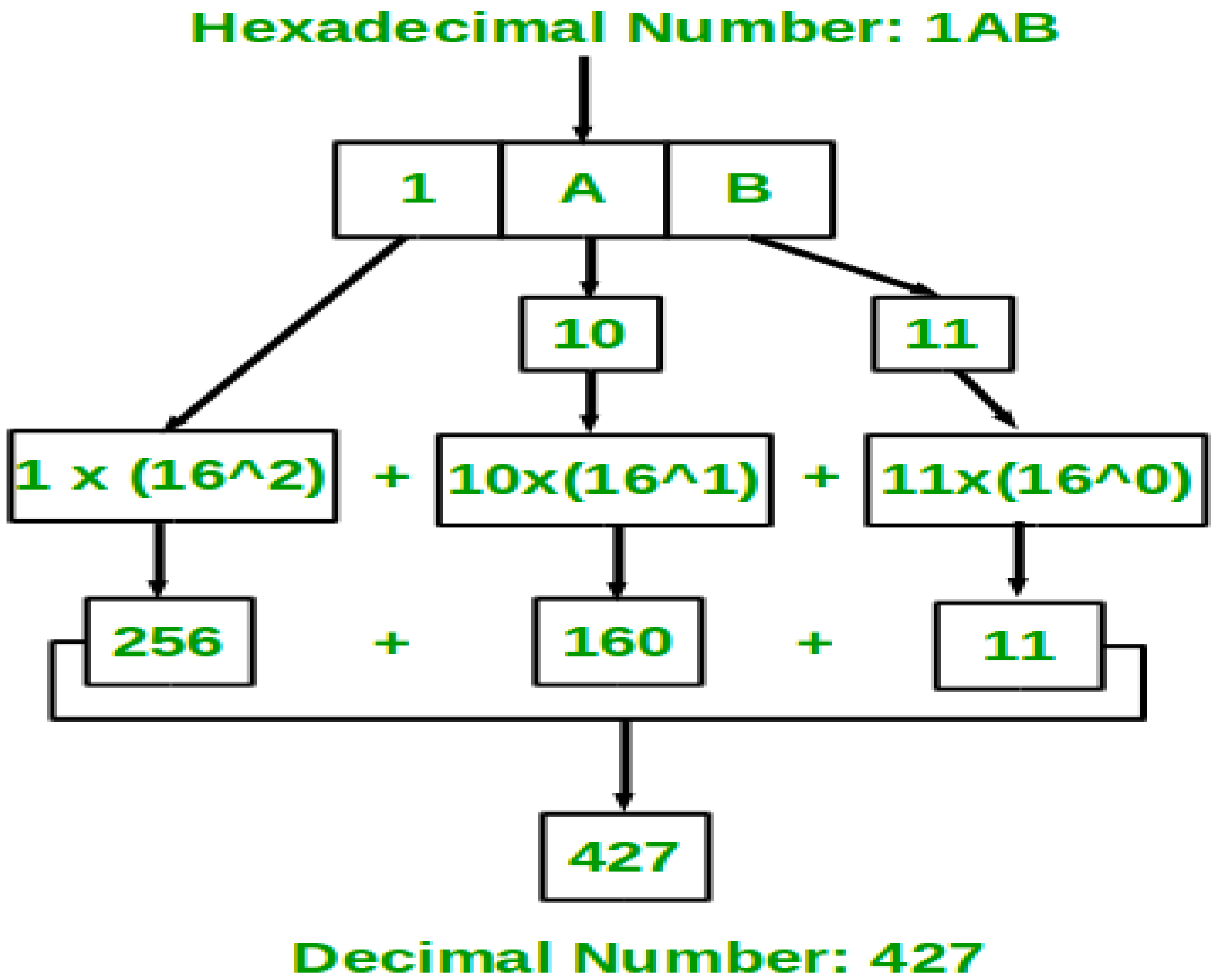
Hexadecimal TO DECIMAL Technique

- Multiply each bit by 16^n, where n is the "weight" of the bit
- The weight is the position of the bit, starting from 0 on the right
- Add the results

EXAMPLE

ABC_{16} =>

$$
\begin{aligned}
C \times 16^0 &= 12 \times 1 = 12 \\
B \times 16^1 &= 11 \times 16 = 176 \\
A \times 16^2 &= 10 \times 256 = 2560 \\
\hline
&\qquad 2748_{10}
\end{aligned}
$$



Hexadecimal Number: 1AB
1
A
B
10
11
1 x (16^2)
+
10x(16^1)
+
11x(16^0)
256
+
160
+
11
427
Decimal Number: 427

Binary to Octal Conversion

Convert the binary number 111110011001_2 to its octal equivalent.

1. Separate the digits of a given binary number into groups from right to left side, each containing 3 bits.

   111 110 011 001

2. Find the equivalent octal number for each group.

111	110	011	001
7	6	3	1

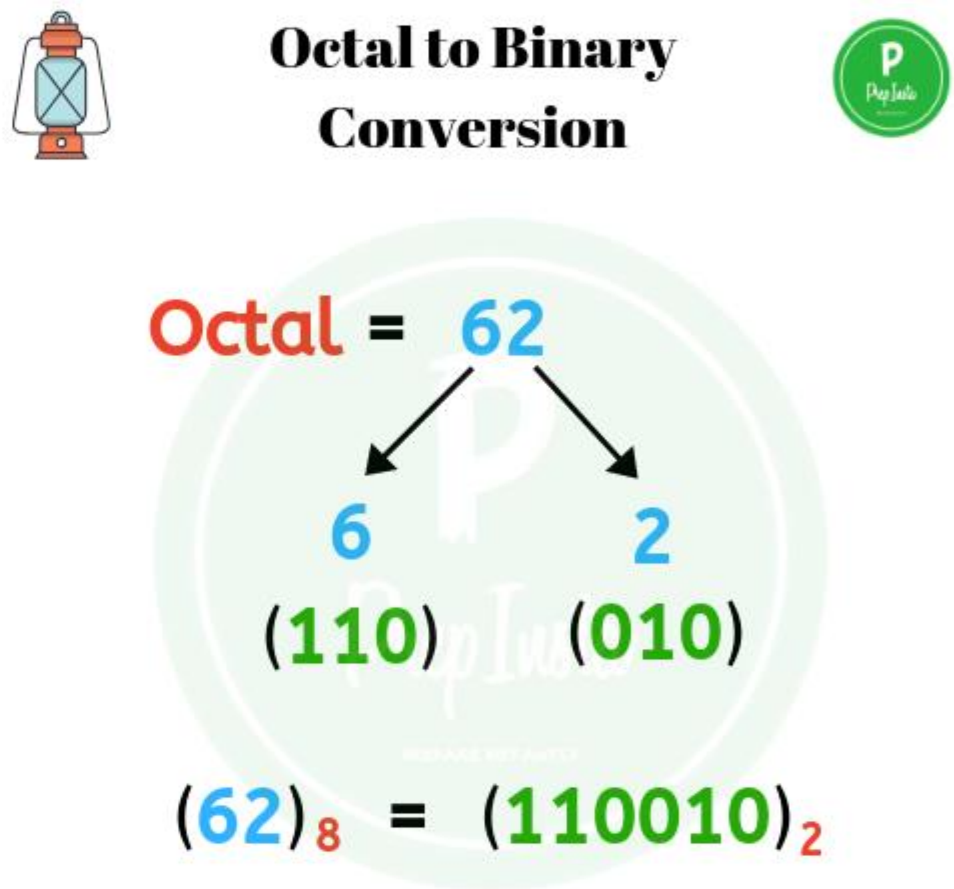
3. Write the all group's octal numbers together, maintaining the group order provides the equivalent octal number for the given binary.

   7631

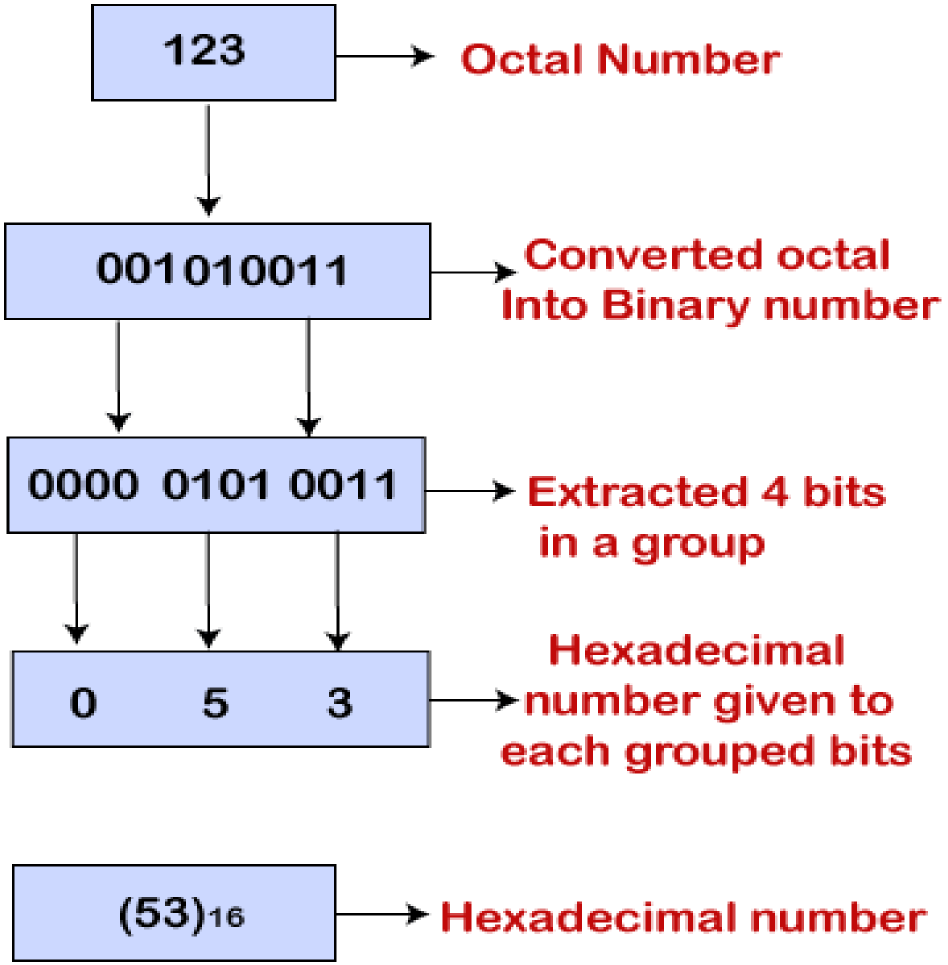
Result

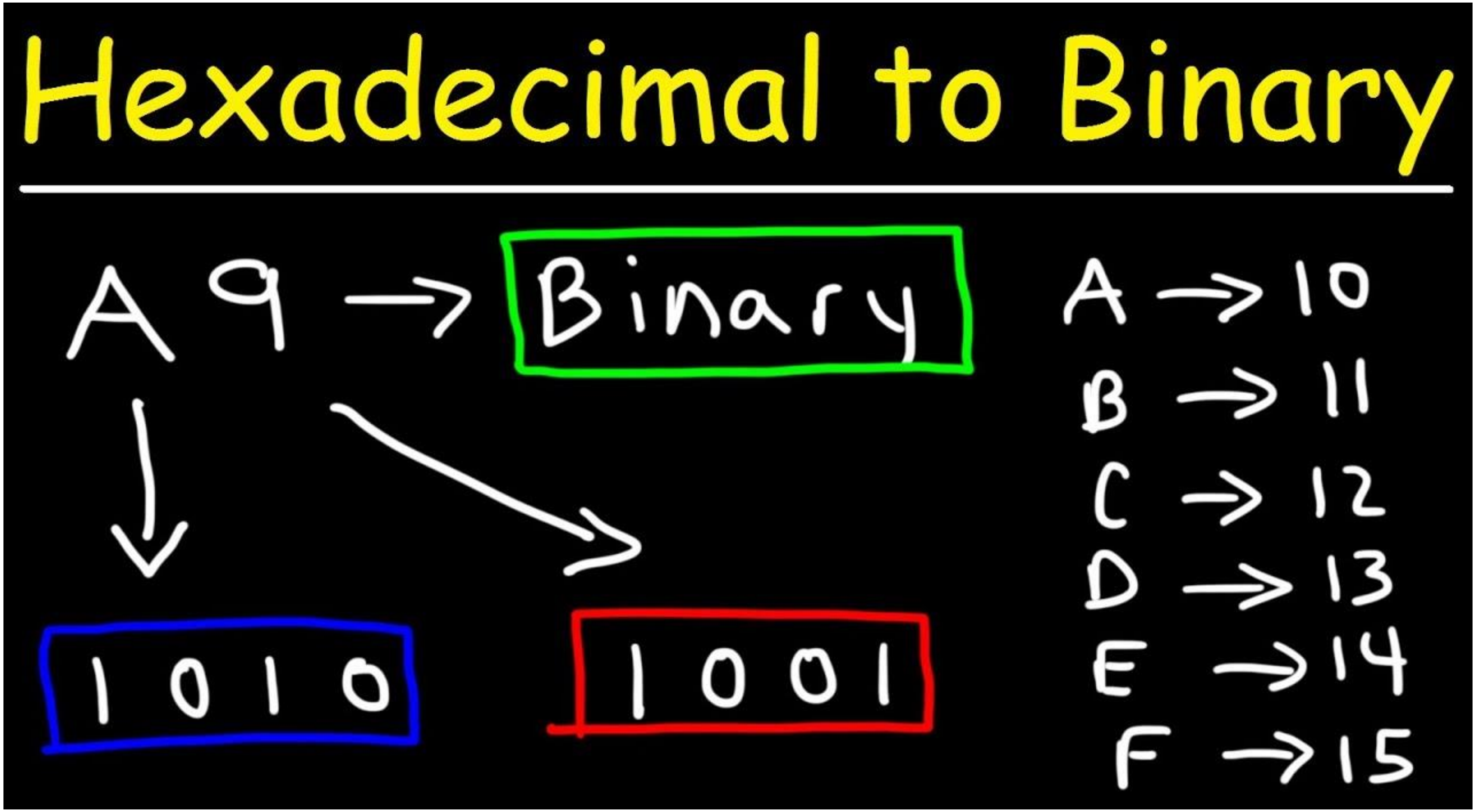
$$111110011001_2 = 7631_8$$

Octal to Binary



Octal To Hexa




Hexadecimal to Binary
A9 → Binary
1010
1001
A → 10
B → 11
C → 12
D → 13
E → 14
F → 15